Abstract—Recently, crowd behavior analysis and abnormal trajectory detection have emerged as a significant research field in the visual surveillance. In this paper, we propose a framework for analyzing the crowd behavior and detecting abnormal trajectories in a structured scene. We employ Fast LDA (latent Dirichlet allocation) algorithm to clustering the trajectories. Our framework is based on this method, and it achieves a faster and more accurate clustering result. In order to obtain the crowd motion pattern, we propose two categories of regions of interest, optimal path and critical regions respectively. We use LOF (local outlier factor) algorithm to detect whether the sample points corresponding to the trajectories are abnormal or not, our framework overcomes the problem whose trajectory length is not uniform. Experimental results illustrate that the proposed framework is effective in motion pattern learning and abnormal trajectory detection.

Keywords-component; Crowd Behavior Analysis; Trajectory clustering; Abnormal Detection; Fast LDA; LOF

I. INTRODUCTION

In recent years crowd behavior analysis and abnormal detection become a challenging topic in the field of crowd management, design of public space, virtual environment, abnormal detection and intelligence environment. In fact, a large number of surveillance devices have been installed in public, such as in markets, subways, gymnasiuims, which collect a great deal of pedestrian trajectory data [1], these huge amounts of data is rich in information, which worthy of further study.

Many methods are applied in unstructured scene previously, the approach adopted in those research focuses on the motion patterns learning and abnormal trajectories detection in unstructured situations [2,3,4]. The analysis of crowd behaviors covers different sub problems such as trajectory clustering and trajectory modeling, for which the goal is to automatically learning motion patterns in scene [5]. The task of trajectory clustering is to assign the trajectories with similar measurements to the same cluster. Trajectory modeling is use of parameterized models to represent each trajectory cluster [6]. The task of abnormal detection is to identify those motion behaviors that are significantly different from other moving objects in the same scene, or to distinguish those behaviors with the probability lower than the motion patterns being found before. The task of behavior prediction is to predict the next movement area or semantic behavior patterns of the observed object based on a priori knowledge and the motion patterns of moving objects.

This paper contribute to crowd behavior analysis and abnormal trajectories detection in a structured scene, because of the influence of the road setting and the obstacles in the structured scene, it is difficult to analyze the motion pattern in the structured scene than in the unstructured scene. The crowd behavior analysis and abnormal trajectories detection in the structured scene can be achieved in our framework. First of all, we preprocess the original trajectory dataset, including trajectory segmentation, calculate the motion parameters and encode sub-trajectories respectively. Secondly, we apply Fast LDA (latent Dirichlet allocation) algorithm [7] to cluster the trajectories. In this way, the rough sub-trajecotry clusters can be acquired. Thirdly, in order to describe the crowd motion patterns, we propose two categories of regions of interest. Eventually, the LOF (local outlier factor) algorithm is adopt to judge whether the sample points of the trajectories are abnormal, which overcome the problem whose trajectory length is not uniform. The framework is minutely described in Fig. 1.

![Figure 1. The framework of crowd analysis and abnormal detection.](image-url)
The remainder of the paper is organized as follows: Section 2 presents the relate work. Section 3 reviews the detail of crowd behavior analysis, which contain trajectory preprocessing, sub-trajectory clustering and motion pattern learning respectively. In section 4, we presents the detail of abnormal trajectories detection. Experiments are presented in section 5.

II. RELATE WORK

Motion patterns learning devoted to build the regular to expression the motion trajectories by the observed data. In [8], Zou et al propose a new approach which constructed c Latent Dirichlet Allocation (CLDA) model to do trajectory clustering. In [9], Zou et al proposed a belief based correlated topic model (BCTM) to do trajectory clustering. Hu et al. [6] clustered the trajectories by using of Dirichlet process mixture model (DPMM) clustering algorithm. Modeled the trajectories by use of a time-sensitive Dirichlet process mixture model (tDPMM).

Abnormal trajectory detection is to identify those motion behaviors that are significantly different from other moving objects in the same scene, or to distinguish those behaviors with the probability lower than the motion patterns being found before. In [10], Claudio Rosito Jung et al proposed to 4-D histogram to detect the abnormal trajectory. In [11], Serhan Cosar proposed an integrated method that incorporates the trajectory-based analysis and pixel-based analysis for abnormal behavior inference.

Behavior prediction is to predict the next movement area or semantic behavior patterns of the observed object based on a priori knowledge and the motion patterns of moving objects. In [12], Josh Jia-Ching Ying et al combined the geographic features and the semantic features to detect the abnormal trajectory.

III. CROWED BEHAVIOR ANALYSIS

In the following, we first preprocess the trajectory dataset, then cluster the sub-trajectory, finally we learn the crowd motion patterns.

A. Preprocessing trajectory dataset

1) Trajectory segmentation:

The preprocessing of trajectories mainly contains three steps in our framework: trajectory segmentation, calculate the motion parameters and encode sub-trajectory.

For charactering spatiotemporal information about the trajectory, we use the sequence of flow vectors to represent the trajectory. In order to stabilize the movement trend of trajectory, a trajectory is segment into a series of sub-trajectory at the turning point. In our case, motion direction can be divided into eight directions: east (0), northeast (1/4π), north (1/2π), northwest (3/4π), west (π), southwest (5/4π), south (3/2π), southeast (7/4π). Then the motion direction of each sample point is discretized to the approximate standard point. If the discretized motion direction of a sample point is different from the previous, these points are regarded as a turning point of the trajectory. Then the trajectories of moving objects are segmented at the turning points.

\[
F_n = \{F'_1, F'_2, ..., F'_m\} \\
F'_i = \{f_1, f_2, ..., f_j\}
\]  

The mathematical expression of the sub-trajectory shown as formula (1), where \(F_n\) is nth trajectory in the trajectory database, \(m\) is the number of sub-trajectories of trajectory \(F_n\), \(j\) is the number of sample points in the ith sub-trajectory, namely the length of the sub-trajectory. The moving direction and motion tendency of each sub-trajectory is relatively stable. The sub-trajectories are easier to researched and analyzed than the whole trajectories.

2) Obtain the motion parameters:

From the above discussion a sub-trajectory in dataset are defined as a sequence of flow vector \(F'_i = \{f_1, f_2, ..., f_j\}\), where, \(f_j = \{x^t, y^t, v^t, \theta^t\}\). original dataset contains only location information \(x^t, y^t\), so the value of velocity and direction of \(f_j\) at time \(t\) should be calculated. So the unknown and useful motion parameters from the original dataset should be extracted for further trajectory processing. The value of velocity \(v^t\) and value of direction \(\theta^t\) of \(f_j\) can be calculated by use of formula (2) and (3).

\[
v^t = \sqrt{(\Delta x)^2 + (\Delta y)^2}
\]

Where

\[
\Delta x = \begin{cases} 
0 & t = 1 \\
x^t - x^{t-1} & t \leq j
\end{cases}
\]

\[
\Delta y = \begin{cases} 
0 & t = 1 \\
y^t - y^{t-1} & t \leq j
\end{cases}
\]

\[
\theta^t = \begin{cases} 
\tan^{-1}(\Delta y) / \Delta x & \text{if } (\Delta x) > 0 \\
\tan^{-1}(\Delta y) / \Delta x & \text{if } (\Delta x) < 0 \text{ and } (\Delta y) \geq 0 \\
\frac{\pi}{2} & \text{if } (\Delta x) < 0 \text{ and } (\Delta y) < 0 \\
\frac{\pi}{2} & \text{if } (\Delta x) = 0 \text{ and } (\Delta y) > 0 \\
0 & \text{if } (\Delta x) = 0 \text{ and } (\Delta y) < 0 \\
0 & \text{if } (\Delta x) = 0 \text{ and } (\Delta y) = 0
\end{cases}
\]

3) Encode sub-trajectories:

LDA [13] usually used to cluster co-occuring words into one topic, it can’t cluster trajectories directly, for further analysis by Fast LDA, it necessary to encode the trajectories and map the sequence of flow vector of trajectories into words in a codebook [3]. We quantized the trajectories according to a codebook. First, we define a codebook to encode the sub-trajectories, we divide the scene image into cells of \(10 \times 10\) pixels, so the space of the scene uniformly quantized into small cells, and quantize the moving direction of each trajectory point into 5 bins, as \(\theta \in \{0,1,2,3,4\}\). The resolution of the space of the scene is supposed to be \(M / (10^2) \times N / (10^2)\) rectangular areas, the size of each rectangular is \(10 \times 10\) pixels. Each rectangle area is represented by the direction of trajectory, 0 indicates without trajectory. Then the directions can be discretized into 5 bins by formula (4)
\[
\theta'_d = \begin{cases} 
1, & \frac{\pi}{4} < \theta^t \leq \frac{3\pi}{4} \\
2, & -\pi < \theta^t \leq \frac{\pi}{4} \\
3, & -\frac{3\pi}{4} < \theta^t \leq -\frac{\pi}{4} \\
4, & \frac{3\pi}{4} < \theta^t \leq \pi \text{ or } -\pi < \theta^t \leq -\frac{3\pi}{4} 
\end{cases}
\]

Then the codebook can be defined as a set of code values:

\[
\text{codebook} = \{0, 1, 2, 3, 4\}
\]

B. Sub-trajectory Clustering

To make the paper independent, we first review the LDA algorithm. In Fig. 2, (a) shows the graphical representation of LDA [13]. In this basic model, shaded circles represent the observed variables, unshaded circles represent latent variables. The arrows indicate dependencies between two variables. Rectangle represents repeated sampling, the number of repetitions in the lower corner. \(\theta\) is a vector of topic, which is follow a Dirichlet distribution which parameter is \(\alpha\). \(Z\) represented a topic, which is a latent variable. \(\omega\) represents words. In our case, \(M\) is number of clusters, \(N\) is the number of trajectory. In Fig. 2, (b) shows the variational distributions for fast model [7]. (c) illustrate fast variational inference of LDA. \(\gamma\) and \(\phi\) are the intermediate parameter. LDA has two model parameters \(\alpha\) and \(\beta\): \(\alpha\) is the parameter of Dirichlet distribution, and \(\beta\) is the set of the discrete distribution parameters for each of \(k\) components over \(V\) words, where \(k\) is the dimensionality of the Dirichlet distribution is assumed known and fixed [13], where \(V\) is the size of the dictionary. Fast LDA algorithm use fast variational inference to estimate the parameters \(\alpha\) and \(\beta\). In our experimental, we randomly initialized \(\alpha\) and \(\beta\).

After encoding, a sub-trajectory is represented by a sequence of words in codebook, so all of sub-trajectory treated as a document, each of the sub-trajectory treated as a word, we use Fast LDA to find out the latent topics, assign the sub-trajectories with similar measurements to the same topic.

C. Crowd pattern learning

In this paper, we propose two kinds of ROI (regions of interest) as crowd motion pattern. The description is as follows:

1) Optimal path:

The optimal path of a trajectory clusters are these scene areas, which contain many sample points of the clusters, corresponding to the region where moving object through with high probability. In our paper, we propose that the optimal path contributes to the region of interest into several grades with different shades of color by formula (6), the more sampling points in this area, the more important the area is.

\[
each block = \begin{cases} 
l & \text{sample points} < a \\
m & a \leq \text{sample points} \leq b \\
h & \text{sample points} > b 
\end{cases}
\]

2) Critical regions of scene:

There has many obstacles and facility in the structured scene, so the trajectories of moving objects will more easily changed than unstructured scene. These regions are significant meaningful to design of public space, public management and public security, and can be called the critical regions of scene. There are two types of critical regions in structured scene:

a) Motion trend change region:

The regions where moving objects change their motion tendency in whole structured scene is named the motion trend change region. Assume that \(F_n\) is \(n\)th trajectory in the trajectory database, then \(F_n\) can be represents with a sequence \(F_n = \{f_1, f_2, ..., f_m\}\), where \(m\) is the number of sample points in the \(n\)th trajectory, \(f_m = \{x_i, y_i\}\). The turning point can be calculated by formula (3) and (4). Then formula (6) are used to count the number of sample points in each regions.

b) Densely region:

The regions where moving objects are often gathered are called densely region. The scene is divided into \(10 \times 10\) pixels rectangle firstly, then number of sample points in each rectangle and in whole scene can be calculated, the formula (7) are used to judge whether the region is densely region. The formula is as

Figure 2. Fast LDA . (a) Graphical representation of LDA [13]; (b) Varitional distributions for fast model[7]; (c) Fast variational inference of LDA.

Figure 3. The framework of crowd analysis and abnormal detection. (a) A sub-trajectory cluster; (b) Scene block; (c)Optimal path.
follows, where, \( N_{\text{rectangle}} \) represent the number of sample points in each rectangle, \( N_{\text{whole scene}} \) represent the number of sample points in whole scene, the D represent the ratio of the two:

\[
D = \frac{N_{\text{rectangle}}}{N_{\text{whole scene}}}
\]  

(7)

IV. ABNORMAL TRAJECTORY DETECTION

A. Testing Trajectories Preprocessing

The testing trajectory dataset needs to be preprocessed before the abnormal trajectory detection for reducing computation. We segmented the original trajectories, calculated the parameters of sub-trajectories and encoded sub-trajectories. Only preprocessing is not enough, each testing sub-trajectory should be assigned to the different cluster according to training clusters. When detecting abnormal trajectories, each testing sub-trajectory just need to compare with the most similar training sub-trajectory cluster rather than the whole, experiments show that it is rapid and effect.

B. Abnormal Trajectory Detecting

The outliers in LOF algorithm [14] are those objects that abnormal in the local scope rather than in global perspective. In traditional methods, objects are only two states: normal and abnormal, so the traditional concept of outliers is simple rigid property. However, LOF algorithm use local outlier factor to describe the degree of the outlier of objects, so it is more applicable to the related applications in real life. LOF algorithm is to identify the abnormal objects by calculating the degree of outlier of each point. The steps of LOF algorithm for abnormal detection is as follows:

1) \( k - \text{distance of an object } p \):

Assuming that \( k \) is a positive integer, \( p \) and \( o \) are sample point on testing trajectory, the distance between \( p \) and object \( o \) is defined as follows, it satisfies two requirements:

(I) at least \( k \) objects \( o' \in D[p] \), it conform that \( d(p, o') \leq d(p, o) \)

(II) at most \( k-1 \) objects \( o' \in D[p] \), it conform that \( d(p, o') < d(p, o) \):

2) \( N_k - \text{distance of object p} \):

The \( k - \text{distance of p} \) has known, then calculate \( N_k - \text{distance of object p} \) on the testing trajectory, the definition of \( N_k - \text{distance} \) is as follows:

\[
N_k - \text{distance}(p) = \{ q | d(p, q) \leq k - \text{distance}(p) \} \quad (8)
\]

3) Reachability distance of object \( p \) and \( o \):

Then we calculate the reachability distance of object \( p \) and \( o \) on the testing trajectory. For any natural number \( k \). The reachability distance of object \( p \) and object \( o \) can be defined by the following formula:

\[
\text{reach} - \text{dist}_k(p, o) = \max\{\text{distance}(o), d(p, o)\} \quad (9)
\]

4) Local reachability density of an object \( p \):

The local reachability density of an object \( p \) on the testing trajectory should be calculated after obtained the value of reachability distance. The definition of local reachability density of \( p \) is as follows:

\[
\text{lr}d_{\text{MinPts}}(p) = \frac{1}{\sum_{o \in N_{\text{MinPts}}(p)} \text{lr}d_{\text{MinPts}}(o)} \quad (10)
\]

5) Local outlier factor of an object \( p \):

Finally, we calculate the local outlier factor of an object \( p \) on the testing trajectory. The local outlier factor of the sample object \( p \) can represent the local anomaly possibility of the sample points, it described as follows:

\[
\text{LOF}_{\text{MinPts}}(p) = \frac{\sum_{o \in N_{\text{MinPts}}(p)} \text{lr}d_{\text{MinPts}}(o)}{|N_{\text{MinPts}}(p)|} \quad (11)
\]

If the local outlier factor of an object \( p \) on the testing trajectory is large than certain value, point \( P \) can be marked as an abnormal sample point. And if most of the sample points of a sub-trajectory are abnormal, this sub-trajectory can be regarded as an abnormal sub-trajectory.

V. EXPERIMENT RESULTS

Experiments are conducted on the pedestrian trajectory database collected from the Edinburgh University [15,16]. This section will demonstrate the results of experiment based on the previous work. The resolution of the space of the scene is \( 640 \times 480 \), according to the encoding sub-trajectory in section 3, the codebook size is \( 32 \times 24 \), so the feature dimensionality of sub-trajectory is 768.

A. Trajectory processing

Figure 4. Trajectory segmentation. (a) and (b) are two original trajectory respectively.

Figure 5. (a) A sub-trajectory before encode; (b) encode result.

The trajectories are segmented into several sub-trajectories at their critical points, which show as Fig. 4, we selected two trajectories randomly, where the yellow point represents the critical point of the trajectory, which is the turning point of the trajectory. Each sub-trajectory \( F_i' \) can be encoded just as Fig. 5, the left figure show a sub-trajectory before encoding, the right figure is corresponding coding results, and the different colors of rectangular represent different directions of trajectory.
B. Sub-trajectory clustering

After trajectory preprocessing, a sub-trajectory training dataset has been obtained. All the sub-trajectories are clustered into several clusters by the Fast LDA clustering algorithm. Fig. 6 illustrates the result of clustering by Fast LAD algorithm, then 36 sub-trajectory clusters in the structured scene can be obtained, each of them represents a kind of crowd motion pattern in a structured scene. Most of the sub-trajectory clusters go along the prescribed path such as both sides of the road or zebra crossing, such as a(1), a(2), a(4), b(1), c(2) and so on, the other sub-trajectory clusters not belong to the prescribed path needs to be study in intensive, such as a(5), b(6), e(4).

Clustering validation has been considered an important indicator for the success of clustering applications [17]. In general, clustering validation consists two classes, external and internal clustering validation [17]. In this paper, we use internal clustering validation to judge the success of the clustering. SSE (sum of the squared errors) and S_Dbw (summation density between and within) [17] are internal clustering validity index respectively, the minimum value of those two index indicates the optimal cluster result. We compared the clustering trajectory validation of our algorithm with those of two clustering algorithms: K-means clustering [18], clustering algorithm based on max-min distance [19]. In experiment, we selected the appropriate parameters of the two comparison algorithms, in order to make the clustering results as precisely as possible.

Table I illustrates comparison of SSE from three approaches, it can be seen that our approach can obtain the minimum value of SSE. Table II shows that comparison of S_Dbw from three approaches, compared with the other two methods, our method...
can obtain the minimum value of $S_{Dbw}$. Therefore our method has better clustering performance than the other two methods.

### TABLE I. SSE (SUM OF THE SQUARED ERRORS)

<table>
<thead>
<tr>
<th>Number of cluster:</th>
<th>26</th>
<th>37</th>
<th>42</th>
<th>46</th>
<th>54</th>
<th>57</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast LDA clustering</td>
<td>7.58</td>
<td>7.09</td>
<td>6.04</td>
<td>6.80</td>
<td>6.91</td>
<td>6.96</td>
</tr>
<tr>
<td>K-means clustering</td>
<td>17.14</td>
<td>12.29</td>
<td>18.60</td>
<td>19.97</td>
<td>20.96</td>
<td>24.53</td>
</tr>
<tr>
<td>max-min distance</td>
<td>18.14</td>
<td>14.29</td>
<td>18.60</td>
<td>19.97</td>
<td>21.96</td>
<td>23.53</td>
</tr>
</tbody>
</table>

### TABLE II. $S_{Dbw}$ (SUMMATION DENSITY BETWEEN AND WITHIN)

<table>
<thead>
<tr>
<th>Number of cluster:</th>
<th>26</th>
<th>37</th>
<th>42</th>
<th>46</th>
<th>54</th>
<th>57</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast LDA clustering</td>
<td>1.36</td>
<td>1.50</td>
<td>1.64</td>
<td>2.17</td>
<td>2.76</td>
<td>2.96</td>
</tr>
<tr>
<td>K-means clustering</td>
<td>4.60</td>
<td>4.94</td>
<td>5.66</td>
<td>5.77</td>
<td>6.51</td>
<td>6.74</td>
</tr>
<tr>
<td>max-min distance</td>
<td>5.60</td>
<td>5.94</td>
<td>6.66</td>
<td>6.77</td>
<td>6.91</td>
<td>7.34</td>
</tr>
</tbody>
</table>

### C. Crowd motion behaviors analysis

![Figure 7. Optimal path of cluster. (a) Cluster34; (b) Optimal path of cluster 34; (c) Cluster11; (d) Optimal path of cluster 11.](image)

![Figure 8. Refining sub-trajectory cluster by optimal path. (a) Cluster14; (b) Optimal path of cluster14; (c) Result of refined.](image)

The optimal path of each sub-trajectory cluster can describe the areas in the structured scene where attractive most pedestrian in this cluster, the optimal path can provide some suggestions for the redesign of the scene. Just as show in Fig.7, many pedestrians in (a) pass through the road not along original design in scene, so the regions of interest will be useful when the scene need to redesign, the designer can set pedestrian crosswalk at those regions in (b). Even though the pedestrians in (c) move along the original design, there are also some areas where trajectories too intensive, more cameras should be set at the regions in (d).

In Fig.8, (a) represents a sub-trajectory cluster, and (b) is the corresponding optimal path. The optimal path of each sub-trajectory cluster is not only helpful to obtain the information of the crowd motion behaviors and structure scene, but also can make a refining to each sub-trajectory cluster. If a sub-trajectory in a sub-trajectory cluster does not pass through the optimal path, it means that the space feature distribution of this sub-trajectory is low probability in its cluster and it can be removed.

Fig. 9 (a) shows the motion trend change regions. They often locate at the road turning. Motion trend change regions illustrate the turning point of trajectory, which can used in the design of public space. Densely regions illustrate the regions whose density is higher than the others. Just as in figure (b). The depths of the color also show the degrees of the attention to the regions. They often located at the building entrance and exit, more cameras should be installed in this place.

### D. Abnormal detection

![Figure 10. Classifying the testing sub-trajectories. (a)-(d) are four testing sub-trajectory.](image)

According to the motion behaviors information of the crowd in a structured scene achieved before, the LOF algorithm is apply to detect the abnormal sub-trajectories. We segmented the testing trajectory dataset into sub-trajectory dataset and then grouped them into several testing sub-trajectory clusters, which can reduce calculation complexity. In Fig.10, we demonstrate the part of the testing trajectory clusters.
In Fig. 11, we demonstrate the results of abnormal detection of 6 clusters. The blue sub-trajectories are normal trajectory and the white sub-trajectories are abnormal trajectory. Experimental results show that LOF algorithm could detect abnormal trajectory in testing trajectory dataset. It could find the objects which away from most other moving objects in a certain period of time. The white trajectory significantly deviate from the blue trajectory. The trajectory of moving object like the white trajectory need further observation and analyze, to ensure the safe and effective of observation scene.

VI. CONCLUSION

In this paper, we proposed a framework to analyze the crowd behaviors and detect abnormal trajectories in a structured scene. Fast LDA algorithm is adopt to clustering sub-trajectory. All of sub-trajectory treated as a document, each of the sub-trajectory treated as a word, and the sub-trajectory clusters can be treated as topics. The clustering result compared with two other clustering algorithms, which proved that our method has good performance. In order to describe the crowd motion patterns of each trajectory clusters, we propose two categories of regions of interest, optimal path and critical regions respectively, which can propose some suggestions for design of public space. In the last part, we use LOF algorithm to detect abnormal trajectory, which overcome the problem that the trajectory length is not uniform. Experimental results demonstrate the good performance of our proposed framework. There are still some improvements in our research, for example, the trajectory prediction, which is an important research direction. In the future research, we will focus on the trajectory prediction.

REFERENCES


