Robust Radial Distortion Estimation Using Good Circular Arcs

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***Abstract*—It is a common problem, radial distortion of off-the-shelf cameras, especially those low-cost ones and wide-angle ones. And the most direct method to judge whether radial distortion occurs in an image is the straightness of those lines in the image, for the straight line in the image should be straight in the ideal image under the pin-hole camera model. In this paper, we present a new line-based approach to eliminate the radial distortion which makes the line distorted in the image. It is based on the fact the straight lines in the real world project to circular arcs under the single parameter division model. Compared with those former line-based methods, the method in this paper work well and be easy to find the good circular arcs, which is valid to eliminate the interference of other curves. Experiments are provided for both synthetic and real images and the results show that our method can remove the radial distortion from images validly and robustly.**

Keywords-radial distortion; division model; circle fitting; good circular arcs;

I. Introduction

An ideal pin-hole camera model is used by most computer vision algorithms, for example, 3D reconstruction, quantitative measurement, recognition and tracking of objects, etc. Its basic assumption is that the 3D straight lines mapping into the image plane are still straight lines. However, in reality small or large amounts of distortion are introduced by most lenses, which bend straight lines in the real world into curves. This may introduce severe problems in the preceding vision algorithms, which making distortion correction is a must.

The imperfection of the lens and the misalignment of the optical system lead to the distortion. And radial distortion is one of kinds of distortion, but it is considered as the predominate one among all possible lens distortion [1 2]. The lens introduces barrel distortion at short focal lengths while it introduces pincushion distortion at longer focal lengths. The polynomial model presented by D.C. Brown [3] has been widely applied for an excellent trade of between complexity and accuracy. Another model widely used lately is the division model presented by Fitzgibbon [4]. And the single-parameter radial distortion model is enough for most lenses and any more elaborated modeling would not help, but also would cause numerical instability [1].

In general, methods for correcting radial distortion can be divided into three categories [5 6]. The first method is multiple view auto-calibration [4 7 8]. No knowledge of the scene and no special pattern is required, but it is not suitable for the distorted image from an unknown source. The second one is point correspondence [1 9]. They identify image points using a known pattern and estimate the distortion parameters as part of the internal parameters of the camera. Hence the results are highly reliable and accurate, but it needs to get multiple images from different views. The last one is plumb-line based method [5 10 11], which assumes straight lines in the real world should be still straight on the image plane. The biggest advantage is that it can correct the distortion only using one image, while the disadvantage is that it needs sufficient lines in the image scene.

What we most expect to the correcting method is easy to remove the distortion automatic and robust from one unknown source image, which is simple and does not need special pattern. Therefore, plumb-based method is the only choice, which can satisfy all the demands. And Wang et al. [10] provides a simple method using the single-parameter division model under the principle that a straight line in the distorted image is a circular arc. The biggest advantage of Wang's method is that it can estimate the distortion center and the parameter of the single-parameter divide model simultaneously. And another advantage of Wang's method is that it can estimate the distortion center and the distortion parameter of the division model only using few straight lines. In principle, three straight lines are enough for computing the distortion center and distortion parameter, and if the distortion center is the image center, it just needs one or two straight lines. Hence, Wang's method can avoid other plumb-line based disadvantage which needs sufficient lines in the image scene. However, Wang's method is not an automatic one that it requires to extract the straight lines manually and the correction results are not robust which vary with the different straight lines.

Then, Bukhari and Dailey proposes an automatic method based on Wang's method which can extract the straight lines automatically and it uses as more straight lines as possible. However, the correcting results are still not robust. And the reason is that they do not solve the problem, selecting out good circular arcs from the curves detected from the distorted image. These line-based methods depend on extraction of long,

This research is partially supported by National Natural Science Foundation of China (No. 61370127, No.61100143, No.61473031, No.61472030), Program for New Century Excellent Talents in University (NCET-13-0659), Fundamental Research Funds for the Central Universities(2014JBZ004), Beijing Higher Education Young Elite Teacher Project (YETP0583). The opinions expressed are solely those of the authors and not the sponsors.
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DOI reference number: 10.18293/DMS2015-021

smoothly curved edges and get thrown easily if a portion of such curves originate from non-linear scene structures [12].

Therefore, in this paper, we analyze all the curves which appear in the distortion images and classify the curve into two kinds, good curves and bad curves. Good curves are the curves projected by the long straight line of the real world, which is crucial to correct the distorted images in the line based methods, especially those automatic ones. Good curves are the edges of the artificial objects, e.g., buildings, signboards, roads, and so on, which are consisted of straight lines. Contrasting to the good curves, bad curves are mainly consisted of three kinds of curves identifying from the edge image. One kind is the curves are both curves either in the distorted image or in the ideal image which means that the curves are still curves, not straight lines, in the ideal images. And one kind is the short curves which might be the curves generating from the lines due to the distortion but is too short to be estimated accurately. Those curves have a common feature that they are too short to be used to estimate the distortion center and the distortion parameter. The last kind the straight lines passing through the distortion center is still straight lines in the distorted images. The reason is that the distortion is mainly radial distortion which makes the pixels move along the radial direction. Therefore, the lines passing through the distortion center are still straight lines either in the distortion image or in the ideal image.

We proposed a non-iterative method to solve this problem selecting out the good curves from all the curves that we extract from the distortion image. And then, we use the good curves to correct the distortion thus avoiding the impact of the bad curves. Our contribution is to make the process fully automatic and robust and it can eliminate the interference of those bad curves very well. The results from the experiments on the synthetic and real image show that the proposed method is simple and valid.

The organization of the remainder of this paper is as follows: Section 2 reviews how to estimate the distortion parameters of the division model and drives an invariant for those points of those lines in the distorted image. Section 3 presents the details of our method. We firstly describe the procedure to select the possible circular arcs from the edge image and estimate their three parameters. Secondly, we find good circular arc using the invariant. Experiments on synthetic and real image are presented in section 4. Section 5 we perform a direct comparison of our method with that of Bukhari-Dailey method [11]. Finally, some conclusions are drawn.

II. ESTIMATE THE PARAMETER OF DIVISION MODEL AND THE INVARIANT

In this section, we review the division model used in this paper and show how to estimate the parameters of this model. Then, we derive an invariant for the points on the curves in the distorted image.

A. Division model

The so-called division model, introduced by Fitzgibbon [4], is

$$\mathrm{r}_u = \frac{r_d}{1+\lambda_1 r_d^2+\lambda_2 r_d^4+\cdots} \quad (1)$$

Where (x_u, y_u) and (x_d, y_d) are the corresponding points of the undistorted image and the distorted image respectively. r_u and r_d are the Euclidean distances of the undistorted point the distorted point to the distortion center (x_0, y_0) . λ_i is the parameter of the model which present the radial distortion.

The advantage of the division model is that it requires fewer distortion parameters than the polynomial model [13 14] for the case of severe distortion [10] and it can estimate the distortion center at the same time. For most cameras, many works [10 11] showed that only the first order radial distortion parameter is sufficient. It can be formulated as:

$$\mathrm{r}_u = \frac{r_d}{1+\lambda_1 r_d^2} \quad (2)$$

And we can write it in the following form

$$\begin{aligned} x_u &= x_0 + \frac{x_d - x_0}{1+\lambda_1 r_d^2} \\ y_u &= y_0 + \frac{y_d - y_0}{1+\lambda_1 r_d^2} \end{aligned} \quad (3)$$

Where, $r_d^2 = x_d^2 + y_d^2$

Another advantage of the division model is that we can easy get the inverse of the single parameter division model. Hence, the pixel coordinates of the distortion image pixels can be presented by the pixel coordinates of the undistortion image pixels. Thus, we can get all the pixel values of the undistortion image using the inverse to find the pixel values of corresponding coordinates in the distortion image. Due to the computing results of the pixel coordinate of the distortion image pixels are not integers, we use simple bilinear interpolation in all of the experiments reported on in this paper.

In order to invert the single-parameter division model [11], we first square the (3) to obtain

$$r_u^2 = \frac{r_d^2}{(1+\lambda_1 r_d^2)^2} \quad (4)$$

Where, $r_u^2 = (x_u - x_0)^2 + (y_u - y_0)^2$

Simplifying the (4), we can get

$$r_d^2 - \frac{1}{\lambda_1 r_u} r_d + \frac{1}{\lambda_1} = 0 \quad (5)$$

For the positive λ_1, when the distortion is pincushion distortion, given $0 < r_u^2 < \frac{1}{4\lambda_1}$, (5) has two positive real roots. We use the smaller one. For negative λ_1, when the distortion is barrel distortion, given any $r_u^2 > 0$, there are two real solution. We use the positive one. Thus, r_d can be presented by r_u. Then, the image coordinates (x_d, y_d) can be obtained as the following formula

$$\begin{aligned} x_d &= x_0 + (\frac{r_d}{r_u})(x_u - x_0) \\ y_d &= y_0 + (\frac{r_d}{r_u})(y_u - y_0) \end{aligned} \quad (6)$$

B. Estimating distortion parameters using the line points from distorted image

Wang et al. [10] has demonstrated that the straight lines in the real world project to circular arcs under the single parameter division model. And Wang et al. use the slope-y-intercept equation from of a line. Similarly, Bukhari and Dailey [11] obtain the same conclusion using the general equation form of a line. For its advantage, we use the general equation form of a line. And it can be written as:

$$ax_u + by_u + \mathrm{c} = 0 \tag{7}$$

Using (3) and (7), we are easy to obtain the circle equation

$$x_d^2 + y_d^2 + Dx_d + Ey_d + \mathrm{F} = 0 \tag{8}$$

Where

$$\begin{gathered} \mathrm{D} = \frac{a}{c\lambda} + 2x_0 \\ E = \frac{b}{c\lambda} + 2y_0 \\ \mathrm{F} = x_d^2 + y_d^2 - \frac{a}{c\lambda}x_0 - \frac{b}{c\lambda}y_0 + \frac{1}{\lambda} \end{gathered} \tag{9}$$

According to the relation of D, E, and F, we obtain the following equation from (9)

$$x_0^2 + y_0^2 + Dx_0 + Ey_0 + \mathrm{F} - \frac{1}{\lambda} = 0 \tag{10}$$

Under (8), we can estimate a group of parameter (D, E, F) by circle fitting method using points belonging to a "straight line" which extracted from the distorted image. Consequently, we can use three groups of parameter $(D_i, E_i, F_i)_{i=1,2,3}$ to compute the coordinates (x_0, y_0) of the distorted center, that is

$$\begin{aligned} (D_1 - D_2)x_0 + (E_1 - E_2)y_0 + (F_1 - F_2) = 0 \\ (D_2 - D_3)x_0 + (E_2 - E_3)y_0 + (F_2 - F_3) = 0 \\ (D_3 - D_1)x_0 + (E_3 - E_1)y_0 + (F_3 - F_1) = 0 \end{aligned} \tag{11}$$

Then we can obtain the radial distortion parameter

$$\frac{1}{\lambda} = x_0^2 + y_0^2 + Dx_0 + Ey_0 + \mathrm{F} \tag{12}$$

C. The invariant for circular arcs in the distorted image

Let (x_c, y_c) and R_c are the center coordinates and radius of a circle by fitting points which belong to a "straight line" extracted from the distorted image. Using (8), we have

$$\begin{gathered} x_c = -\frac{D}{2} \\ y_c = -\frac{E}{2} \\ R_c = \sqrt{\frac{D^2+E^2+4F}{4}} \end{gathered} \tag{13}$$

Algorithm 1: Choosing good circle arcs and estimate parameters.

Input:

Arcs parameters set $\{(\mathbf{x_c}, \mathbf{y_c})_i\}$ and $\mathbf{R_c^i}$ $\mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \cdots, \mathbf{NumberOfArcs}$

[min, max] are the range of $\mathbf{C_{lg}}$ and T is the interval for counting the number of $\left(\mathbf{width}/_{\mathbf{2}}, \mathbf{height}/_{\mathbf{2}}\right)$ is the image center

Output:

$\boldsymbol{\lambda}, \boldsymbol{x_0}, \mathbf{y_0}$ are the distortion parameters

Begin:

Compute the constant using Eq. (14) for all candidate circular arcs

If $\mathbf{NumberOfArcs} \geq \mathbf{3}$

Divide the [min, max] into equal intervals, each adjacent interval overlaps half interval, and for each interval, count the number of constant those fall into the interval. Find the interval with maximal constant values support. Then, compute the mean value or mid-value of those constant as the ideal value denoted as Cm. Chose the circular arcs which constant values fall into the area[Cm-T/2,Cm+T/2] as good circular arcs

End

Estimate $\boldsymbol{\lambda}, \boldsymbol{x_0}, \mathbf{y_0}$ using good circular arcs

End

After reformulating the equation (12), we obtain

$$\frac{1}{\lambda} = (x_0 - x_c)^2 + (y_0 - y_c)^2 - R_c^2 \tag{14}$$

From the equation (14), we can know that the difference of square of Euclidian- distance of the center of the distorted image and the center of the fitting circular arcs with square of radius of the fitting circular arcs is an invariant to all the "straight lines" in the distorted image. And we can use this invariant to find the good circular arcs, which is valid to eliminate the interference of other curves.

III. ROBUST ESTIMATION METHOD

In this section, we describe the details of our automatic and robust method to correct the radial distortion using the single parameter division model.

A. The main procedure of our method

To sum up, the whole process to remove the radial distortion includes four steps and is presented as follows:

1. Extract image edges in the distorted image for detecting circular arcs.

2. Identify circular arcs from the image edges and estimate their parameters for each arc, the coordinates of the circular arc center and the circular arcs radius are included.

3. Find good circular arcs for computing the parameter of radial distortion.

4. Compute the distortion parameter and correct the distorted image using the single parameter division model.

For the first step, we employ the Canny Detector [15] to extract the image edges and link adjacent edge pixels remaining

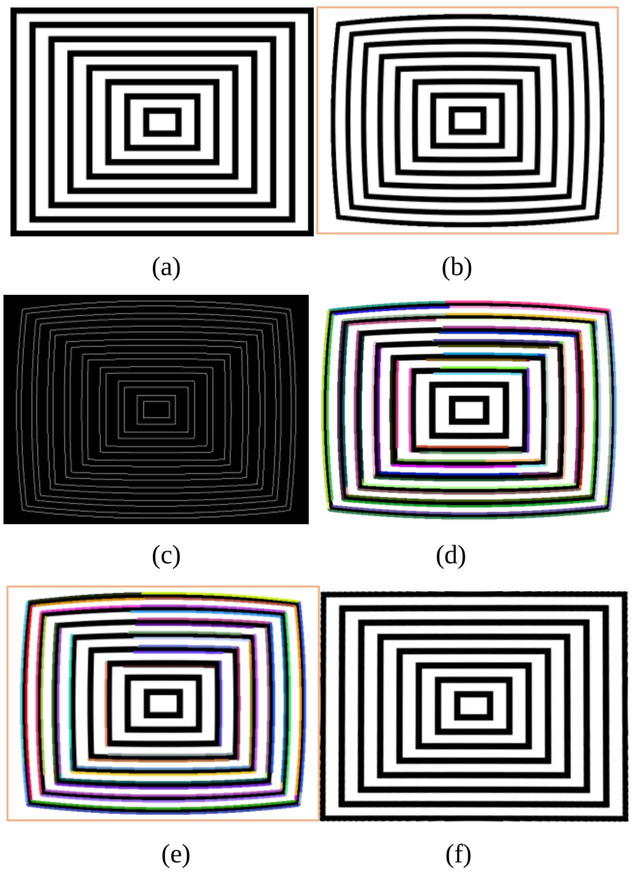


Figure 1. Undistorted processes on the synthetic image. (a) The ideal synthetic image with no distortion. (b) The synthetic image with distortion parameter $\lambda_{true} = -1.0 \times 10^{-6}$. (c) The result of canny edge detection. (d) The synthetic image with detected circular arcs. (e) The synthetic image with detected good circular arcs. (f) Undistorted image of the synthetic image. The size of the interval is 0.6.

contours [11]. And we discard those short contours and remain the long ones for more reliable information they provide and when fitting the short and therefore straight edges in circles, the estimated parameters are known to be unstable [16]. Hence, a threshold is set. If the contours whose number of pixels is less than the threshold are discarded for they are too short to be used. For the second step, a modified RANSAC method is used to detect circular arcs not overlapping with other arcs in the same contour that have more support [11]. The termination criterion of the modified algorithm is that to stop once the probability that an arc of minimal length has not yet been found is small. In order to refine the estimated circle parameters, we use the Levenberg-Marquardt (LM) iterative nonlinear least squares method [17] to estimate them. In the next step, we introduce a voting process to filter the good circular arcs, which is presented in detail in the flowing subsection. We can select out the good circle arcs from the circular arcs that extracted from the distortion image in the previous step. In the last step, (11) and (12) are used to compute the distortion parameter and distortion center by using the circle parameters which are found in the preceding step. And then, (6) is used to get the undistortion image. Thus, we implement a process of correcting distorted images automatically. In order to get robust results, the third step is very important.

B. *Choosing the good circular arcs*

According to the (14), we can know that good circular arcs have the same constant. Inversely, we can screen out the good circular arcs by a voting process for the constant with maximal support inspired by Hough transformation [18]. And the details are presented in Algorithm 1. Due to the distortion center is unknown, we use the image center to replace it in the algorithm 1 for that the distortion center is nearby of the image center in most of distorted images [17]. Therefore, compute results have deviation with the ideal value but still fall into a small range nearby the ideal value, showed in Fig. 1. And we also conclude from (14) that the ideal constant is reciprocal of the distortion parameter. In reality, the distortion parameter is very small, generally less than 0.00001, hence the constant is very large. So we transform the constant into a special logarithm domain. The transformation relation is presented as follows:

$$C_{lg} = \begin{cases} \lg(c) & if, c > 0 \\ 0 & if, c = 0 \\ -\lg(-c) & if, c < 0 \end{cases} \tag{15}$$

$$\mathrm{c} = \frac{1}{\lambda} \tag{16}$$

Where c is the compute results of (14), and C_{lg} is value in the special logarithm domain. And the region $-15 \le C_{lg} \le 15$ is enough, that is $|\lambda| \ge 10^{-15}$.

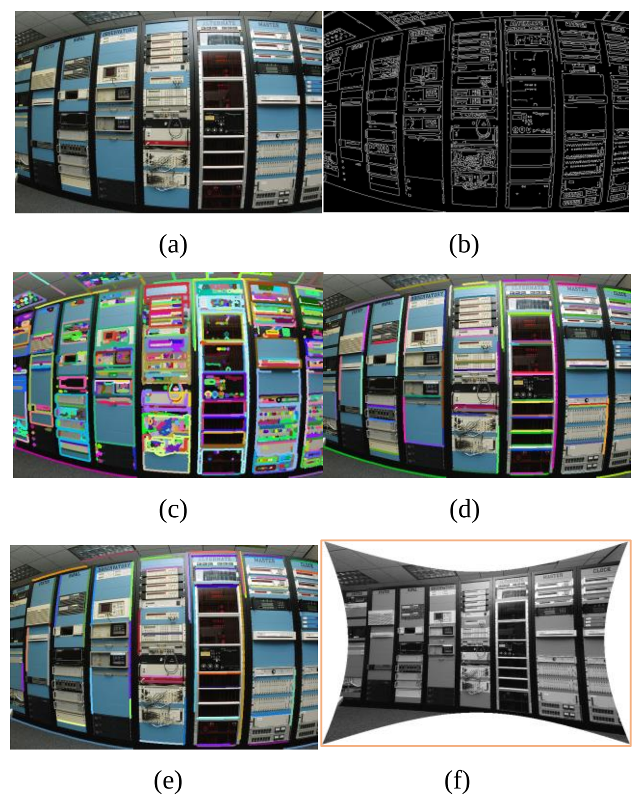
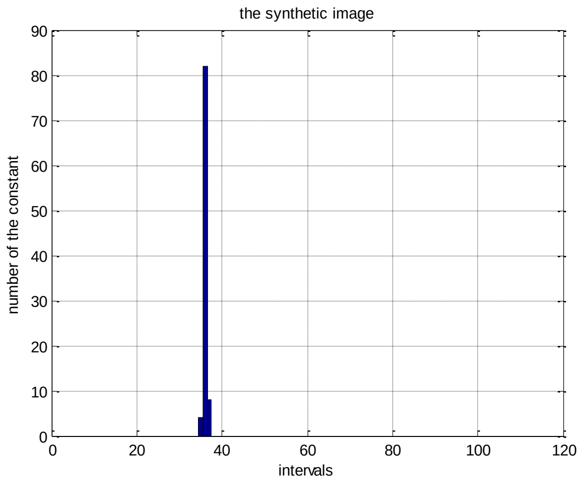
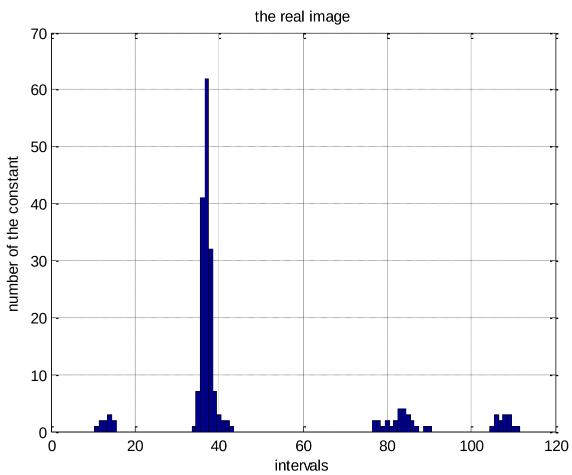


Figure 2. Undistorted processes on the real image. (a) The real distorted image. (b)The result of canny edge detection of the real image. (c) The result of liked contours. (d) The real image with detected circular arcs. (e) The real image with detected good circular arcs. (f) Undistorted image of the real image. The size of the interval is 0.6.



(a)



(b)

Figure 3. The distribution of the constant values in special logarithm space. (a) Distribution of the constant values of the synthetic image. (b) Distribution of the constant values of the real image.

IV. EXPERIMENTS AND RESULT

In this section, we present a detailed study of our method on synthetic and real image data. We distort the same original image (see Fig. 1(a)) for all the synthetic images by using particular ground truth values of the distortion parameters and the division model. And the size of all the synthetic images is 640x480. The minimum length of the detected lines is 100 pixels.

A. *Experiments on synthetic and real images*

In order to test and verify our method, we performed experiments on synthetic and real images. The synthetic image with known distortion parameter $\lambda_{true} = -1.0 \times 10^{-6}$ and distortion center (320, 240) is shown in the Fig. 1(b). And Fig. 2(a) is a real image of 800x531 obtained from a publicly available database [19].

The Fig. 1(c) is the canny detection result of the synthetic image and in the Fig. 1(d), the detected curves are represented by using a different color to identify them. The synthetic image has sufficient "straight lines" which are the basis for correcting the distortion. The Fig. 1(e) shows the results of the good arcs which are selected out from the detected curves presented in the Fig. 1(d). The correcting result of the synthetic image is presented in the Fig. 1(f). It is obviously that the proposed method can undistort the synthetic image very well.

Fig. 2 are the undistortion process of the real image. Similarly, the total process have five parts which are described in detail in the previous chapter. The results of canny detection, contours linking, identifying circular arcs and finding good circular arcs are presented in the Fig. 2(b)-Fig. 2(e) respectively. Fig. 2(f) that the proposed method also has a good correcting result on the real image.

As observed in Fig. 3(a) the constant values are gathered into a small range, about two or three intervals, while in the Fig. 3(b), most of the constant values are into a small range but some of them fall into other intervals. It is mainly because the circular arcs detected from the synthetic image are all good ones, while the circular arcs detected from the real image include the bad ones, as explained in Section 3. The constant values of good and bad circular arcs are distributed into different intervals, hence we are easy to separate them in the special logarithm space.

B. *The influence of the size of interval*

As described in the proposed method, the special logarithm is divided into equal size intervals and the number of constant values are counted which fall into the same interval. Therefore, the size of the interval has a significant impact on the good circular arcs we selected. In this subsection, we discuss the influence of the size of the interval on the synthetic and real image. The size of the intervals is set to 0.2, 0.4, 0.6, 0.8 and 1.0 respectively in the experiments. The synthetic image we use is the distorted image of $\lambda_{true} = -1.0 \times 10^{-6}$, showed in the Fig.1 (b). And the real image we use is the Fig. 2(a). The results of the experiments are presented in the Fig. 4.

In the Fig. 4, first row are distributions of the constant values of the synthetic image under the different size of intervals and the second row are the corresponding synthetic undistorted images. From the third, we can know that the distribution of constant values are nearly the same concentrated into only a small range, two or three intervals, which illustrates that the assumption of the proposed method is right. The reason why the size of the intervals has little influence on the synthetic image is that the circular arcs detecting from the image are very good. Hence, correcting results in the second row are all good. Third row: distribution of the constant values of the real image under the different size of intervals. And the fourth row: corresponding undistorted real images. From correcting results in the fourth row, we can know that the results turn a little bad when the size of intervals is too big or too small. We can know the reasons from the third row that the constants values fall into more intervals when the size of the interval is too small. So the interval we use to select the good circular cannot include all the good circular arcs we want. While the interval would contain the bad ones when the size is too big. From the results, the size of the interval should not too big or too small and should be set between 0.4 and 0.8. In additional experiments, we set the size is 0.6.

C. *Experiments on iamge with varying distortion centers*

In Algorithm 1 we compute the constant values using the image center instead of the distortion center, hence we test our method on synthetic images with different distortion center. In the following experiment, to find the influence of the distortion center, the centers of the synthetic images are: (320, 240), (290, 210), (260, 180) and (230, 150). And the distortion parameter is $\lambda_{true} = -1.0 \times 10^{-6}$.

The results are presented in Fig. 5. In the first row, from left to right, distorted images with different centers are shown. The second row illustrates the distribution of constant values of the distorted images with different distortion center. As observed, when the distance of distortion center with the image center is greater, the distribution of the constant values is more scattered, that is, the constant values are distributed in more intervals, but the maximum is still in the same interval. Hence, the proposed method is still suitable for the case that the distortion center is not in the center of the image. And the undistorted results in the third also demonstrate that the distortion is good to eliminate.

D. *Experiments on image with different distortion parameter*

In order to verify the performance of the proposed method, we take a series experiments with varied distortion parameter lambda. The distortion centers of the synthetic images fix at $(x_0,\ y_0) = (320,\ 240)$. And the distortion parameters λ_i are respectively: -1.0×10^{-5} , -1.0×10^{-6} , -1.0×10^{-7} and -1.0×10^{-8}.

Fig. 6 shows some results of the experiments, first row are the distorted images at different levels of lambda, second row are the distributions of the constant values of the synthetic images with different distortion parameter, and the third row are corresponding undistorted images. From the row of the Fig. 6, the absolute value of the distortion parameter is bigger. The distortion is more serious. And in general, the value of the distortion parameter is still very small, even though the distortion is very serious. When $\lambda = -1.0 \times 10^{-8}$, the distortion is very small which cannot be watched out with human eyes. Hence, range of the special logarithm, we set the previous chapter, is enough. From the second row of Fig. 6, with the varying of the distortion parameter, the distribution of the constant values is different which are concentrated into different intervals.

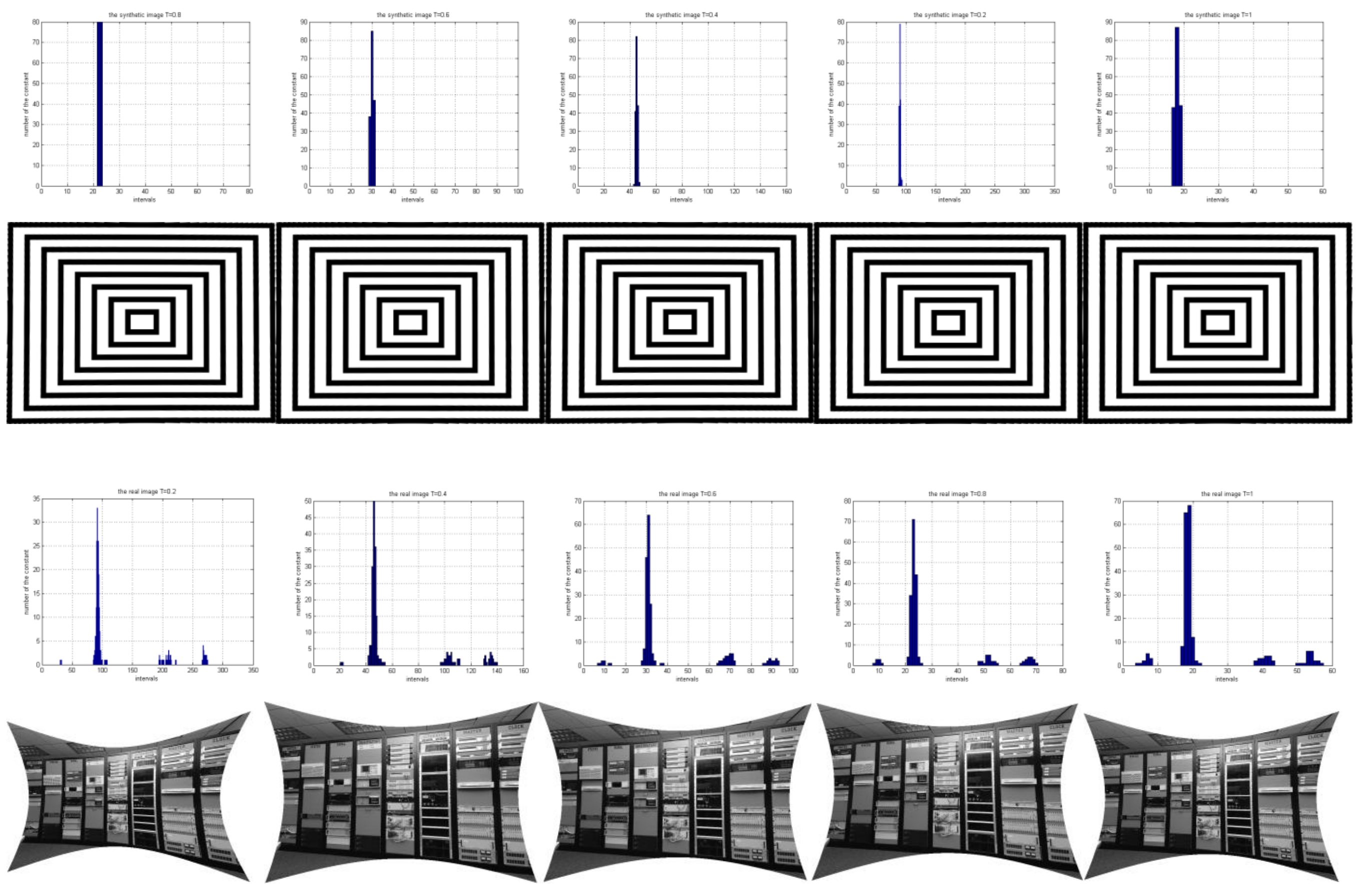


Figure 4. Undistortion of synthetic image and real image in different size of interval. Each column presents the image with the same interval size, and the size of intervals are 0.2, 0.4, 0.6, 0.8 and 1.0. The first row and the second row are the results of the synthetic images, and the third row and the forth row are the results of the real images. First row: distribution of the constant values of synthetic image under the different size of intervals. Second row: corresponding undistorted synthetic images. Third row: distribution of the constant values of real image under the different size of intervals. And fourth row: corresponding undistorted real images.

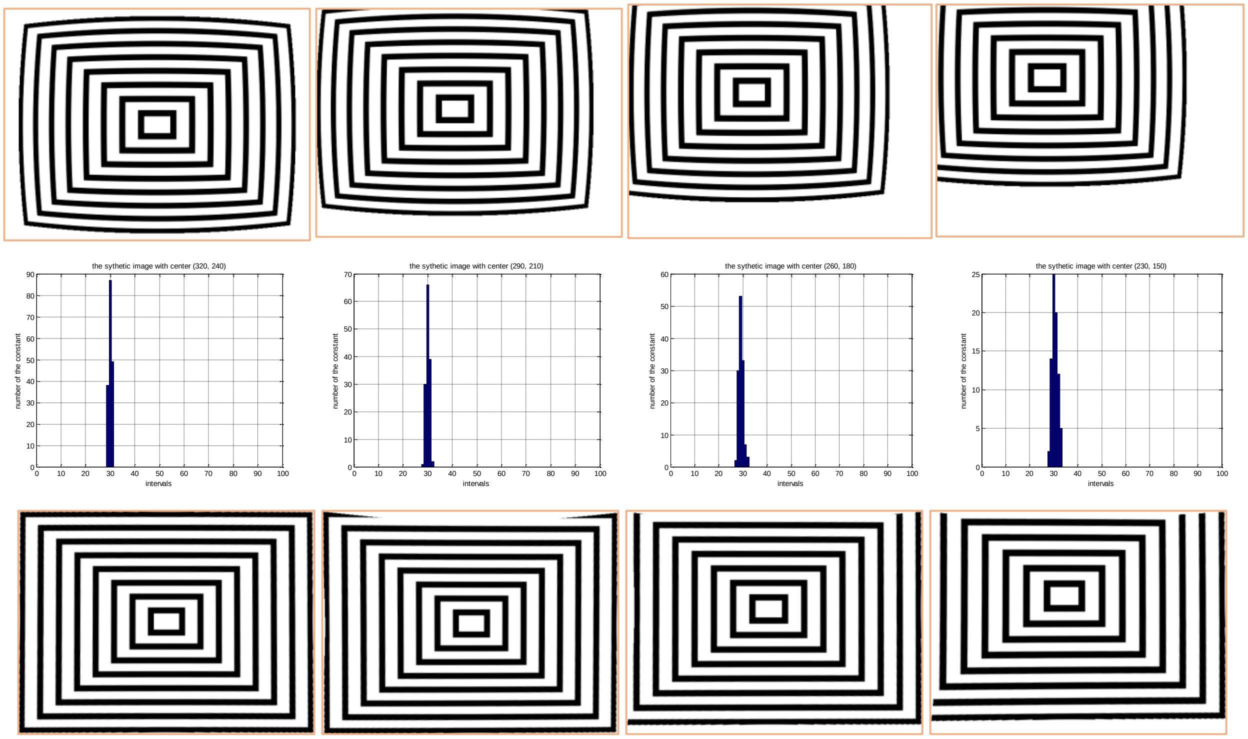


Figure 5. Correcting synthetic images with different distortion center. The image size is 640x480 and the columns from left to right the corresponding centers are: (320,240), (290,210), (260,180) and (230,150). First row: distorted images with different distortion centers. Second row: the distribution of the constant values of the corresponding distorted images. Third row: corresponding undistorted images.

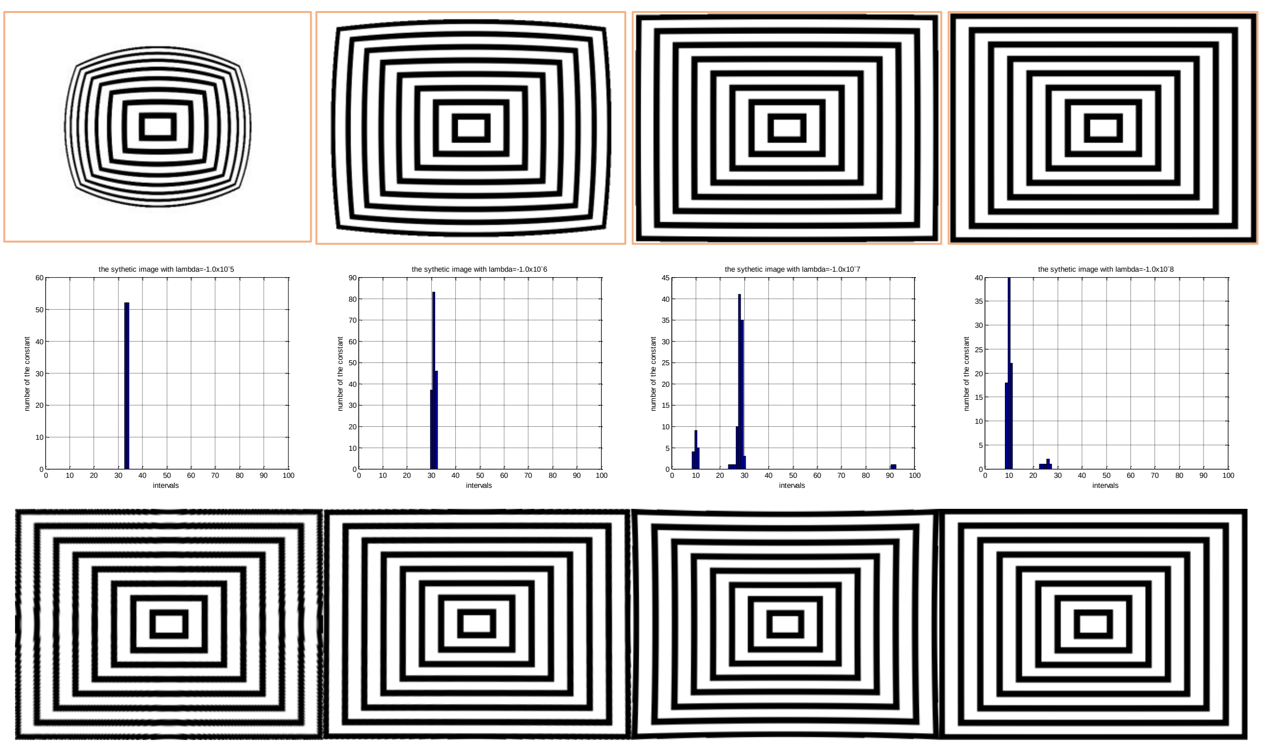


Figure 6. Undistortion of synthetic images with different distortion parameters. Image size is 640x480 and distortion center is (320,240). First row: distorted images at different levels of lambda. Second row: distribution of the constant values of the synthetic images with different distortion parameter. Third row: corresponding undistorted images

Figure 7. Lens distortion correction for a real image:(a) Original image, (b) detected candidate arcs, (c) choose the good arcs using the Bukhari-Dailey method, (d) choose the good arcs using the proposed method, (e) undistorted image using the Bukhari-Dailey method, and (f) undistorted image using the proposed method.

When the distortion parameter is small, there are several constant values falling into other intervals which are far away from the intervals most constant values concentrated. The reason is that there is some bad circular arcs in the curves we extracting from the edge image and the source of the bad circular arcs are almost straight curves which parameter cannot be estimated accurately. From correcting results in the third row, we can know that the results of the small distortion images are not as well as that of the serous ones. It mainly the reason is that the circular parameter cannot be estimated accurately for the length of the curve is relatively small with the full circle.

V. Comparison with Bukheri-Dailey method

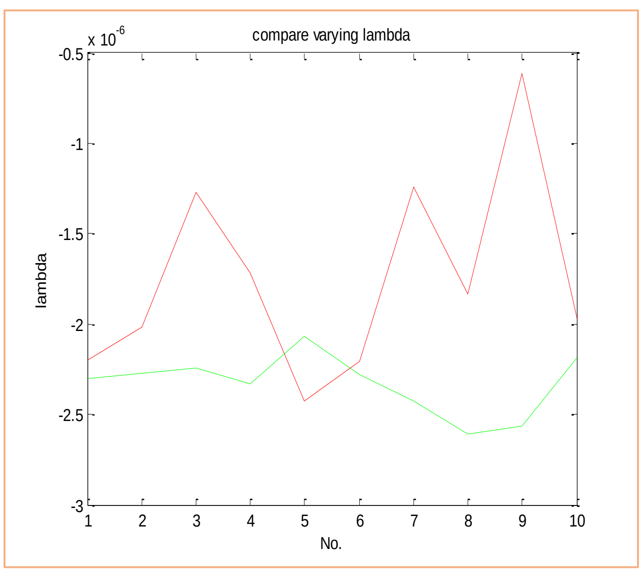
Fig. 7 presents the results of a real image with 422x311, from a publicly available database, shown in Fig. 7(a). And Fig. 7(b) shows the arcs detected results which are identified with different colors. Fig. 7(c) presents the results of the good arcs selected by the Bukhari-Dailey method, whereas Fig. 7(d) presents the results of good arcs selected by the proposed the bad circular arcs, while the proposed method can. For instance, the curves of the window and door in the image are bad arcs witch should be removed and the short curves is also should be eliminated. Fig. 7(e) and Fig. 7(f) are corresponding correcting results. It is obvious that the proposed method gets a better result when there are bad circular arcs in the distorted image.

Fig. 8 presents the results for another real image of 720x515, from a publicly available database [20], which has many short straight line in the distorted image, see Fig. 8(a) and Fig. 8(b). The estimation results of distortion parameter in Fig. 8(c) indicate that the varying of distortion parameter of the proposed method is smaller. And the estimation location of distortion centers of the proposed method is more concentrated than that of the Bukhari-Dailey method. Hence, the proposed method is more robust.

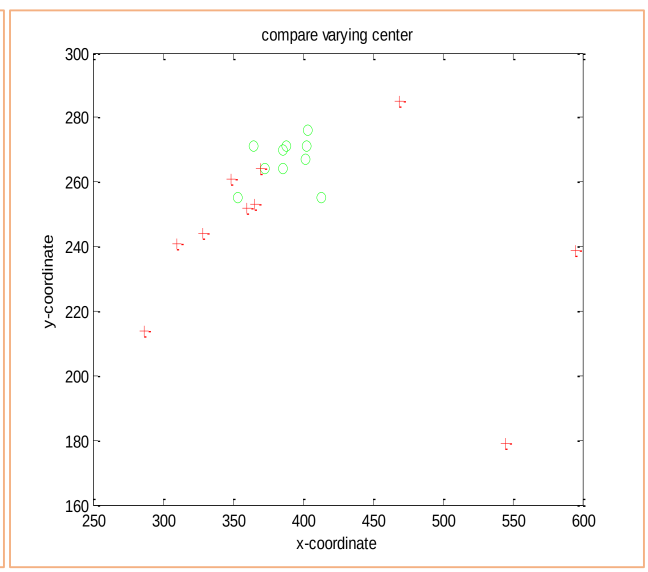
Table 1 shows some quantitative results which illustrate the time costs for different number of arcs. Minimum pixel number of arcs, average number of arcs and average CPU time for Fig. 8(a) are computed using the Bukhari-Dailey method and the

(a)

(b)



(c)



(d)

Figure 8. Compare the variation of distortion parameter in 10 runs:(a) Original image, (b) detected candidate arcs, (c)magnitude of lambda (the green line are results of the proposed method while the red line are that of the Bukhari-Dailey method). (d) location of distortion center(the green symbol “o” presented the distortion center using the proposed method and the red symbol “+” are the distortion center using the Bukhari-Dailey method)

TABLE I.

Fig. 8(a)	Minimum length of arcs	180	160	140	120	100	80
The Bukheri-Dailey	Average number of arcs	12.2	17.7	23.9	37.3	59	110.6
	Average CPU time(MS)	1357.3	1835.5	3495.5	4480.3	4504.5	4691.9
The Proposed method	Average number of arcs	11.2	18.4	23.2	37.9	61.8	109.4
	Average CPU time (MS)	7.3	7.6	7.8	10.9	15.2	19.8

proposed method respectively. Each test runs 10 times, and the CPU time includes selecting good circular arc and estimating the distortion parameter. As observed, it is obviously the proposed method just takes a little time which is far less than that of the Bukhari-Dailey method. It is primarily because the proposed method is simple and non-iterative, while the Bukhari-Dailey method requires an iterative process. With increasing number of the circular arcs, the time costs of the pro posed method increases but are still very small, while that of Bukhari-Dailey method increases fast. The proposed method of choosing good circular arcs is faster than that of the Bukhari-Dailey.

VI. CONCLUSION

In this paper, a method to identify good circular arcs from the detected distorted curves which are vital to line-based method of estimation of distortion parameters, especially the fully automatic ones. It is based on that the constant values are the same for all the good circular arcs. The algorithm is simple, robust and non-iterative. Once good circular arcs are determined, the parameters of distortion can be estimated accurately. Therefore, the proposed method recognizing the good circular arcs and only using the good circular arcs to correct the radial distortion can eliminate the interference of the bad curves. We have presented a variety of experiments on synthetic and real images which show that the proposed method allows removing the radial distortion automatically and robustly.

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